Showing and telling: Primary students' outcomes in data representation and interpretation

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Students, in triads in a near-classroom environment, were video-taped as they worked on interpreting and representing supplied data. Their responses were categorised using the SOLO taxonomy. Representation skills varied from copying out some of the data to relating two variables graphically; interpretation skills varied similarly. Moreover, there appeared to be connections between the two skills. The study also considered the nature and effectiveness of the collaboration which took place within groups.

The performance of students working in collaborative environments has received considerable attention in the past 20 or so years, and such studies have often involved mathematical tasks. In the past decade the study of chance and data has become an explicit part of the mathematics curriculum across all grades in most states of Australia. This study arose out of earlier research which was initially concerned with the abilities of students in finding and representing relationships in data and which later came to consider, in addition, the nature of the collaboration that takes place if they are working on such tasks in groups.

Background to the Current Study

The "data cards protocol," which is described briefly in the next section, was first used to investigate students' functioning in the chance and data area of the curriculum in the work of Watson, Collis, Callingham and Moritz (1995). The researchers in that report studied students working in small groups in an actual classroom, using a single roving video camera to record what happened, before analysing the graphs and tables produced by the students for evidence of different levels of understanding. They found that there were two three-step cycles of functioning: one where the student acknowledges associations in data sets with increasing sophistication, and the second involving the justification of those associations. Later Lidster, Chick and Watson (1997) used the data cards protocol with both individuals and groups of three students removed from the classroom, to consider the way that hypothesising and data representation together contribute to the ability to interpret the data and confirm hypotheses. Again a video camera recorded the actions of the students, to gain evidence of what they hypothesised and what they produced as output. In the process of considering the four groups and twelve individuals used in that study it became apparent that one of the groups was particularly productive: the group members' hypothesising and interpretation skills were sophisticated, they grappled creatively with the difficulties of representing the data to justify their hypotheses, and, most notably, their collaborative interactions were highly efficacious. This prompted a closer study of that group and the remaining three in order to examine aspects of collaboration and determine their effect on cognitive outcomes (see Chick & Watson, 1997, for a case study of the single group, and Watson & Chick, 1997, for consideration of all four groups plus some groups involved in a different protocol).

The 1997 studies suggested that working in groups had some advantages over working alone for the data cards protocol, particularly for the noteworthy group, but it was acknowledged that the environment in which the groups worked was "ideal" in a certain sense. Each group worked on the protocol away from the regular classroom, with the interviewer present providing occasional cues. This may well have influenced the levels of concentration and cooperation. Certainly, it is not obvious that such productivity and understanding would result from group work conducted in a standard classroom, notwithstanding some fairly positive results achieved in the 1995 study. This question provided some of the motivation for the present study. In the light of the growing emphasis on chance and data in the curriculum, a second question concerned the natural level of understanding that the students actually bring to the protocol without receiving specific instruction in data representation and interpretation. The final issue of interest, following from the first two studies, concerns the connection between different aspects of statistical cognition — in particular, whether or not there is a relationship between the ability to interpret and the ability to represent the data.

In general the verdict on the effect of collaborative settings on outcomes associated with mathematical problem-solving tasks is still open (e.g., Davidson, 1985; Good, Mulryan & McCaslin, 1992). On one hand, Lehrer and Romberg (1992) were impressed with the data modelling which occurred when Grade 5 children worked as a group of ten in one instance and as a pair in another. Stacey (1992), on the other hand, had some reservations about the outcomes when Grade 9 students worked in groups of two or three on algebraic problem solving. She felt that students often selected the easiest or simplest approach to a problem, with detrimental consequences when such choices were incorrect.

It should be highlighted that the data cards protocol used in the current study is an exceptionally open-ended task. Unlike most mathematical problem-solving activities, which typically have a discernible end-point so that students know when they have achieved "the answer," the data cards protocol allows students to explore a variety of questions using numerous possible approaches. From some students' perspectives this open-endedness may be somewhat disconcerting.

Method

The data cards protocol on which the students worked (described in more detail in Watson, *et al.*, 1995, or Watson & Callingham, 1997) involved the consideration of a set of 16 cards, each bearing name, age, weight, weekly fast food consumption, favourite activity and eye colour information for a fictitious but realistic juvenile. The students were asked to look for and attempt to show any interesting features of the data.

The 27 students who participated in this study were from a Grade 5/6 class at a suburban primary school. Their teacher informed the researchers that the average abilities of the students were generally lower than other classes in her experience. She was asked to divide the class into groups of three, and she claimed that the students had successfully undertaken collaborative activities in mathematics in similar groupings. They had not, however, participated in any data gathering, representation or interpretation activities with this teacher.

There were five sessions in all, conducted over three weeks. To facilitate video-taping of the individual groups, the first three sessions were run twice, back to back, firstly with five of the nine groups and then with the remaining four. These sessions were 45 minutes long and the researchers who conducted them (the first author and another researcher) reminded the students of the expectation to work in groups. The first session was introductory and allowed an exploratory examination of the data cards, with a view to identifying any interesting features. At the beginning of the second session students were shown a variety of different ways of representing data, such as bar graphs, pie charts, tables and scatter plots. The students were encouraged to prepare a poster to display their findings, a task which continued into the third session.

As one focus of the research was to determine the "natural" statistical abilities of a "standard" class prior to specific instruction, the researchers were deliberately vague in describing the task and their expectations to students. Students were merely asked to try to find interesting things, followed up by questions like "How might you show that?" with an overarching instruction that the group was expected to come up with a poster to show what had been found. Showing the students some examples of graphs at the beginning of the second session was intended to provide a clue and a focus for approaches that could be taken if desired. To reinforce the potential value of graphical representation the researchers supplied graph paper as well as blank paper.

After the completion of the posters, a fourth session was held with the whole class, where members of each group were given the opportunity to present and explain their poster to their peers. During the fifth and final session the second author — who had not been present for the earlier sessions — interviewed the students individually, asking them questions about their posters and their feelings about group work. One purpose was to determine the students' abilities to explain their approaches and understanding to someone who had not seen their initial working.

All sessions were video taped, and where the students were working in their groups there was a camera focussed on each group. The tapes were transcribed and both the transcriptions and the original video tapes, plus interim rough work produced by students and copies of the group posters, were used to analyse the students' collaborative behaviour and statistical outcomes. This categorisation will be described below. Where judgements have been made about the levels of statistical outcomes attained these were made by the authors independently, and when initial interpretations disagreed discussion took place until consensus was reached. A similar approach was taken for categorising the levels of collaboration exhibited by each group, but involved the first author and the transcriber. Most disagreements were a result of slightly different interpretations of definitions; excluding these, agreement was reached without discussion being necessary in over 80% of cases.

Analysis

There were two aspects to the students' statistical results: (a) Their ability to observe properties and interpret the data, which will be referred to as "interpretation," and (b) their skill in depicting the data through, for example, graphical or tabular means, which will be referred to as "representation." Both aspects were analysed using the Structure of Observed Learning Outcomes (SOLO) model (e.g., Biggs & Collis, 1991) which considers *modes* of functioning and classifies students' responses according to the level of complexity observed and whether or not relevant information is considered. In this case the students are generally functioning in the concrete symbolic mode of the SOLO taxonomy. Three main levels are distinguished within this mode: unistructural, multistructural, and relational. Each level represents an increasing order of complexity, as indicated in Table 1; moreover these levels are identifiable for both interpreting and representation. There is also a prestructural level preceding unistructural; it is likely that this indicates functioning more typical of the ikonic mode. Finally, there is a level following relational, namely extended abstract (EA). This is characterised by responses which are more general, not necessarily restricting themselves to the specific details of the data or problem which elicited them. It indicates that the response is moving into the next mode of functioning, namely the formal mode.

A wide variety of collaborative — and non-collaborative — behaviours were observed, to the extent that it may have been reasonable to define nine categories for collaboration, one for each group. Some groups had episodes in which two or all members collaborated effectively, and then their behaviour would degenerate for a time. Other groups worked reasonably harmoniously, but with no extensive development of relevant ideas. A member of one particular group was unwilling to engage in the task, which had a deleterious effect on that group, although he later became motivated about an aspect of the work on which he then worked alone. At times, particularly when the students were producing graphs for the posters, there was a tendency for a group to appear to be three individuals, rather than a collaborative group. Nevertheless, despite the range of collaborative behaviours it was possible to classify the groups into one of four categories.

- C1 There was little or no collaboration between group members.
- C2 There was substantial disruption and/or antagonism within the group; however, some non-trivial collaboration took place between pairs or even the whole group.
- C3 There were no overt clashes between group members, notwithstanding the occasional negative undercurrent; some sharing of ideas occurred, but not in a very productive way.
- C4 Considerable useful collaboration took place; furthermore, the social environment was generally harmonious.

Note that our interest is in collaboration which directly affects mathematical outcomes, so that a group which socialised amiably without discussing the task would not be regarded as having collaborated productively. Some reasons for the disparate behaviour of the groups are proposed in the discussion.

Table 1.
SOLO levels used to classify students' responses in interpretation and representation.

SOLO level	General Characteristics	Interpretation	Representation
Pre-structural/ Ikonic (P/IK)	Uses no relevant information; may refuse to engage in the task, or attempts to deal with it ikonically	Introduces patently irrelevant ideas to the situation	Does not consider the data at all; e.g., colours in a graph produced by someone else, or decorates the poster
Unistructural (U)	Uses only one relevant aspect of the information	Considers individual aspects of the data; e.g., suggests reasons for the results on a single card at a time	Depicts individual aspects of the data set or a subset; e.g., records some or all data values in a table with no attempt at aggregation
Multistructural (M)	Several aspects of relevant information are used, often in sequential fashion rather than with more complicated connections	Considers all the data, but considers only one variable; e.g., notes that more people like television than any other activity	Can represent a single variable aspect of all the data; e.g., by drawing a bar graph of eye colour
Relational (R)	Integrated understanding of the relationships among various aspects of the information	Proposes cause and effect relationships between two variables; e.g. between fast food consumption and weight	Can depict a two variable relationship; e.g., by drawing a scatter graph

Results

During the first session, when the students were exploring the data and recording some of the things that interested them, very few came up with the idea of using a graphical representation to depict the data. Some wrote out information from a subset of the cards, others recorded some summative information in tables, while others verbalised their observations. At the beginning of the second session, however, when one of the researchers showed various graphs and tables, nearly all students indicated that they had seen and produced a bar graph on some occasion, with many indicating familiarity with pie charts, line graphs, and 2×2 tables as well. The least familiar type of representation was a scattergram. The researcher's presentation of different kinds of data representation was enough to prompt most of the students to use graphical approaches to the data for their posters. It is not clear whether this shift in strategy was because the students realised the value of such representations for summarising data and illustrating their attributes or because they judged that since the researcher seemed to value them — as shown by her demonstration — then that was what they ought to do.

As indicated in the previous section, students were classified on three criteria: the SOLO level of their interpretation, the SOLO level of their data representation, and the level of collaboration exhibited by their group. The results for the 27 students are shown in Table 2. Entries in the table are individual students, identified by their group number; the four narrow shaded vertical columns (within the representation columns) represent the different categories of collaboration, ranging from C1 on the left to C4. Thus, for example, the last entry in the table is one of the three students from Group 6, a group whose collaborative behaviour was assessed as being in category C3, while the student's SOLO levels were extended abstract for interpretation and relational for representation.

Table 2.
Students' outcomes and levels of collaboration on the data cards protocol.

		REPRESENTATION																			
		Nil				Prestructural/ Ikonic (P/IK)				Unistructural (U)				Multistructural (M)				Relational (R)			
		C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
INTERPRETING	U					G7															
	M						G3 G5								G3 G3 G5 G9	G2 G4	G8 G8				
	R			G1				G1				G1		G7 G7	G5 G9	G2			G9 G9	G2 G6 G6	G8
	EA																			G4 G4 G6	

Collaboration categories: C1 (darkest shading), C2, C3 (lighter shading), C4 (unshaded)

It must be pointed out that the *highest* level of functioning which was observed was the one which was recorded. Some of the students ventured an interpretation based on a few cards — a unistructural response — before responding multistructurally by putting forward more general analyses. A number of students made no statistical contribution to the final poster, instead doing headings or decorations. These students have been classified as having a prestructural SOLO level of representation, or nil in the case where nothing was done at all. The assessment of the students' representations was based on their work during the sessions and not just on what was on the final posters. One of the students in Group 6 had completed a scattergram (or equivalent) which was omitted from the final poster because it was similar to but judged as messier than one of the other student's efforts, while the third student in the group started a scattergram but was absent for the third session and so did not complete it.

As can be seen from Table 2, half of those who actually produced a representation of the data, did so at the same SOLO level as their interpretation, with either both outcomes being multistructural or both relational. The remaining thirteen students were able to interpret data at a higher SOLO level than they were able to represent it. Perhaps not surprisingly no students gave interpretations which had a SOLO level lower than that of their representations. It might be expected that valid and complex interpretations can be made even when there are shortcomings in the representation. On the other hand, if an appropriate representational form has been done well it cannot help making the interpretation easier. The power of suitable representations is such that they describe the data in an obvious way, and lend themselves to having interpretations made at a level commensurate with the depiction. During the researcher's illustration of a variety of graphical forms at the beginning of the second session, many students indicated an understanding of what the graphs and tables were telling them, yet very few chose to use the more complicated forms, such as scattergrams, to illustrate relationships such as that between weight and fast food consumption in their own analyses. It is thus proposed that there is a cognitive difference between being able to interpret data presented in a carefully selected representational form and being able to choose the appropriate form for oneself.

The data in Table 2 further support this hypothesis. Two of the students who produced multistructural representations but gave relational interpretations came from groups (Groups 2 and 9) in which another member had produced a relational representation. These students were able to interpret the graphs made by their colleagues, either by direct examination and interpretation of the graph, or as a consequence of discussions that took place among the students in the course of the collaborative group work. The remaining students who had relational interpretations and lower level representations, seemed to have based their interpretations on an extensive study of the cards, and thus proposed some relationships in the data despite the fact that no one in the group had depicted the data in a relational way.

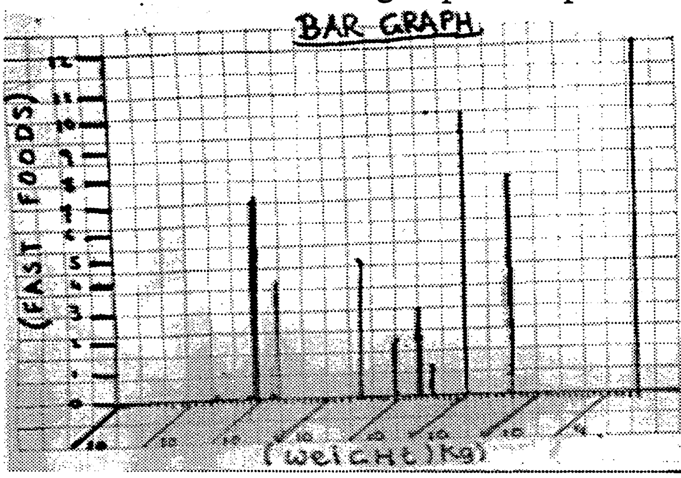

Figure 1. Two variable graph completed by a student in Group 6.

The remaining three students, whose interpretation level is indicated as extended abstract in Table 2, displayed a level of interpretative behaviour which went beyond the others. Two of them had completed a scattergram (or similar), while the third was a member of the same group as one of those who did so. All were able to discuss the fact that the data did not fit a theory of increased weight with increased fast food as well as they had supposed. Having had no explicit experience with variability in data sets one of them was prepared to reject his hypothesis on the basis of the lack of a more uniform linear trend in the data (see Figure 1). They were the only students of the 27 to indicate that the representation was allowing them to *prove* (or, in this case, cast doubt on) an hypothesis, rather than merely showing what had already been observed. One commented that "It doesn't show what I wanted it to show," which led to a discussion about other influential variables, such as age, and as a result he produced a weight versus age graph which was also included on the poster. For most other students the graphs seemed only to give a concrete representation of what they already knew. The SOLO level of these three interpretations is verging on extended abstract, because of the students' ability to interpret the chosen data beyond just making a summative observation.

Only one student interpreted the data at a unistructural level by recounting various facts from individual cards. She seemed unable to interpret the graphs produced by her colleagues in the final interview. Her own representation level was prestructural because she just wrote down data from a few of the cards that interested her.

Finally, it appears that the nature of the collaborative behaviours in the groups has little influence on the outcomes. Students with lower level outcomes came from groups whose remaining members did quite well, and this was true of all collaboration categories. Certainly it would be inappropriate to make any generalisations based on the data as analysed thus far.

Table 3 summarises the different kinds of representational form used by the students on their final posters. Some students produced two graphs for the final poster while others did not have any thereon for a variety of reasons. This explains why there may be more or fewer than three representational forms for a given group.

Table 3.
The different representational forms used by the students on their posters.

Representation type (approximately in order of increasing SOLO level, where P=Prestructural, U=Unistructural, M=Multistructural, R=Relational)	Students producing that graph representation (identified by group number)	Total
Table (partial information) (P)	G5, G7, G7*	3
Table (all aspects of all data) (U)	G1	1
Bar graph (partial information) (M)	G9	1
Bar graph (frequency) (M)	G2, G2, G3, G3, G4, G4, G7, G8, G9, G9, G9	11
Pie chart (M)	G8	1
Line graph (but equivalent to a bar graph) (M)	G5	1
Bar graph with grouped data (weights) (M)	G9	1
Bar graph with an extra variable (girls v boys) (R)	G5, G9	2
Pie chart with information about an extra variable (R)	G8	1
Scattergram, but with lines rather than dots (such as shown in Figure 1) (R)	G6, G6	2
Scattergram (R)	G4	1
Two comparative scattergrams (girls v boys) (R)	G2, G4	2

* This table consisted of a very specific subset of the data — those who watch television — and was adjudged to be a multistructural representation.

Discussion

It should be pointed out that there was a range of performance levels even within the various SOLO categories. Some students, for example, made relational connections but were not convincing in their discussion of them, while others considered more general issues such as whether the results of the data analysis apply to other people. The ability to employ SOLO classifications for both the interpreting activity which took place and the representations (graphical or tabular) produced is very useful in this setting. The fact that half of the students performed at the same level on both aspects of the task indicates they can combine understandings in two related domains consistently. The remaining students were able to interpret the data at a higher level than their representation skills, suggesting further research is needed to determine reasons for this.

As mentioned earlier, collaborative setting seems to have no discernible effect. Discrepancies such as the fact that individuals in some groups do not do as well as others in the same group point to the need for a closer consideration of the other factors involved in collaboration in the classroom setting. It is certainly true that these groups did not work together as well as the three boys who formed the focus of the study of Chick and Watson (1997). Indeed, the researchers had a definite feeling that the groups often behaved more as three individuals than a genuinely collaborative team.

It should be reiterated that the groups were formed by the teacher, and that the students had worked collaboratively on mathematical tasks before this study was conducted. Nevertheless, the groups formed for the data cards protocol did not appear to be the same as those used on a problem-solving task observed by the researchers prior to the data cards sessions. This was backed up by a comment made by one of the students during the course of the activity. All the groups were mixed, by gender in all cases and by school grade in most. The authors feel that this probably had a strong influence on the outcomes of the study, and that it would have been possible to form "better" groups. It must be emphasised, however, that one of the purposes of this study was to consider the realities of the classroom, and that teachers may not always be able to choose the best groups in the first instance or, indeed, in any instance if there are constraints to be taken into account. In this case, the researchers had imposed the requirement that the groups have three members, as this duplicated the group size used in the earlier studies and it also made viable the video taping of individual groups. Furthermore, the teacher apparently consciously decided that she wanted the students to work in mixed-gender groups, as this was observed during other visits to the classroom.

In this study the students did not readily bring their previous knowledge and experience to bear on this problem. As mentioned in the results section, despite having encountered and produced various representational forms prior to the sessions involving the data cards protocol, virtually all students were unable to transfer that experience to this situation without being cued by the researchers. This points to the need for teachers to make these connections explicit in the upper primary years.

There is scope for further research on the data obtained from this class of students. As mentioned earlier a wide variety of collaborative behaviours was observed, not all of them constructive. It would be interesting to determine whether or not useful ideas were lost because of detrimental behaviour, and to examine the students' perceptions of the collaboration which took place in their groups.

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